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Discrete-time approximation of multivariable Continuous-time delays systems

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Abstract— **Many works are related to the analysis and control of either continuous or else discrete time-delay systems. However, the discretization of continuous time-delay systems has not been extensively studied. In this work, sampled-data time-delay systems with internal and external point delays are described by approximate discrete time-delay systems in the discrete domain. Those approximate discrete systems allow the hybrid control of time-delay systems. Two Numerical examples complete the paper, showing the correctness of the discretization process.**

Keywords— **Time-delay systems; Multivariable control systems; Discretization; Simulation.**

I. INTRODUCTION

Systems with Delay abound in the world. They appear in various contexts such as biological, ecological, economic, social, and engineering systems. Typical examples of time-delay systems are communication networks, chemical processes, teleoperation systems, bio-systems, underwater vehicles and so on [17,19,2]. In many physical, industrial and engineering processes, delays occur due to the finite capabilities of information processing and data transmission among various parts of the system. Delays can arise as well from inherent physical phenomena such as mass transport flow or recycling. Also, they can be byproducts of computational delays and can be constant or time-varying, known or unknown, deterministic or stochastic, depending on the system under consideration. In all of these cases, the time-delay factors have, by and large, counteracting effects on the system behavior which in most cases lead to poor performance. Therefore, the subject of time-delay systems has been investigated in the form of functional differential equations over the past three decades [22,2,3,8].The engineering literature dealing with time-delayed systems are very extensive. Most of the approaches proposed so far deal with linear time-delay control systems and, in particular, with the stability analysis and behavior of such systems with constant and/or uncertain time-delays [12,9,10]. However, it should be mentioned that conventional numerical techniques, such as the Euler and Runge-Kutta methods, have been employed in order to obtain a sampled-data representation of the original continuous-time delay-free system [5,13].All of these approaches require a very small time step in order to be deemed accurate, however this may not be the case in control applications where large sampling periods are inevitably introduced due to physical and technical limitations. Furthermore, the use of too many integration steps may cause unacceptable integration times and even the excessive accumulation of errors.

With the rapid advances in the large-scale integration of semi-conductor devices and the resulting availability of inexpensive computers, there is renewed interest in the discrete-time approximation of continuous-time multivariable systems. Such models have applications in the digital simulation of these systems as well as in the identification of the system through samples of the input- output data. Further applications of these methods are possible in digital adaptive control and computer control of complex processes [6].Systems including time delay, due to the system dynamics, are widely present in the industry which imposes a lot of constraints that make the control and computer programming of such system difficult [7,14].Then the control of a system with a time delay is generally difficult; due to the constraints imposed by the time delay. These constraints can cause performance deterioration that leads the process to instability especially when operating in closed loop [1,23,24,25].Most physical systems, a macroscopic point of view, are continuous. In modern control systems, information is digitally processed which requires sampling signals. One speaks in this case of sampled or discrete systems.

For this reason we need the discretization of continuous linear delay systems. Other works are devoted to the study of discrete delay systems [11,20,16,15].The objective of this paper is extending the ideas in the just cited references in order to analyze in this paper, we consider two different methods for obtaining the discrete-time delay approximation. These are:

(i) State-transition method.

(ii) Method based on the trapezoidal rule for integration.

The paper is organized as follows: The next section discusses discretization of systems with external point delay; section 3 discretization of system with internal and external point delay; Section 4 provides a numerical examples; Section 5 comparing between the two methods and a conclusion section closes the paper.

II. DISCRETIZATION OF SYSTEMS WITH EXTERNAL POINT DELAY

In a digital computer, time cannot flow continuously as it is perceived in the physical world. The time is defined on a discrete set of times, which are separated by a regular time interval known by one sampling period. It is therefore necessary to define new mathematical tools adapted to discrete time, to represent the sampled signals and systems and adapt tools and methods for automatic analog (continuous time) in the design of digital controllers.

Then our problem may be stated as the determination of a discrete time approximation corresponding to the following state-space equations:

$$\begin{cases} \dot{x}(t) = A_0 x(t) + A_1 x(t-h) + Bu(t) \\ y(t) = cx(t) \end{cases} \quad (1)$$

Where:

$A_0 x(t)$: Original Term state.

$A_1 x(t-h)$: Delayed Term state.

With $h = qT$: a multiple delay the sampling period is an integer q

T : The sampling period assumed chosen suitably.

$x, u \text{ and } y$ Respectively are the state vector, the vector and the control vector output.

A_0, A_1, B and C are matrices of suitable dimensions.

In the development of the discrete-time models, we have to assume a suitable sampling interval, denoted by T. A suitable criterion for the choice of T is that wT be less than 0.5, where w is the magnitude of the Eigen value of farthest from the origin of the s-plane [6]. Among the main concerns of the sampling was one that is not to lose information in the temporal discretization of the continuous signal. For this to be possible, a condition to be fulfilled is that the signal that we must sample has a finite spectral width, it is called a spectrum of low-pass type. Since the spectral width of such a signal is defined by the interval [0, F_{max}], where f_{max} is the highest frequency present in the frequency spectrum of this signal. This condition arises from the phenomenon of aliasing. In reality, all physically realizable analog signals spectral width is "great", although it is necessarily finite energy. This may arise, for example, the presence of additive noise or interference with amplitude spectra is not negligible at high frequencies. In other words, f_{max} is very large which necessarily leads to the choice of an even greater f_e. If there is aliasing, it is not possible to return to the original signal spectrum. In this case, the operation changes the sampling characteristics of the input signal. Thus, if we do not want to lose information relative to the signal which is sampled, we must always satisfy the condition: ($Fe > 2F_{max}$). Condition better known by Shannon's theorem .

A. *State-transition method*

The description of the system dynamics in the form of differential equations is retained throughout the analysis and design. In fact, if a subsystem is characterized by a transfer function it is often necessary to convert the transfer functions equations in order to proceed by state-space methods. The state-transition matrix which describes how the state x (t) of the system at some time t evolves into the state x(T) at some other time T. For time-invariant systems, the state-transition matrix is the matrix exponential function, which is easily calculated. For most time-varying systems, however, the state-transition matrix, although known to exist, cannot be expressed in terms of simple functions .This is the most well known method. If we assume that the input is allowed to vary only at the sampling instants and held constant during each sampling interval, we obtain the following equation:

$$x(k+1) = \psi_1 x(k) + \psi_2 x(k-q) + \psi_3 u(\mathrm{k}) \quad (2)$$

$$\forall t \in \left[kT, (k+1)\mathrm{T}\right] \;, \qquad u(t) = u(k) \quad (3)$$

Where, for notational convenience, *x (kT)* has been represented by *x (k),* and

$$\psi_1 = e^{A_0 T} \quad (4)$$

$$\psi_2 = e^{A_1 T} \quad (5)$$

$$\psi_3 = \int_0^T e^{A_0 t}\, Bdt + \int_0^T e^{A_1 t}\, Bdt \quad (6)$$

The matrices ψ_1, ψ_2 and ψ_3 can be conveniently calculated on a digital computer using the following series:

$$\psi_1 = I + A_0 T + \frac{A_0^2 T^2}{2!} + \frac{A_0^3 T^3}{3!} + \ldots\ldots \quad (7)$$

$$\psi_2 = I + A_1 T + \frac{A_1^2 T^2}{2!} + \frac{A_1^3 T^3}{3!} + \ldots\ldots \quad (8)$$

$$\psi_3 = \left(2I + \frac{A_0 T}{2!} + \frac{A_1 T}{2!} + \frac{A_0^2 T^2}{3!} + \frac{A_1^2 T^2}{3!} + \ldots. \right) TB \quad (9)$$

B. *Method based on trapezoidal rule of integration*

The determination of a discrete model of a continuous linear or nonlinear process can be envisaged by a transformation on the functional matrix that characterizes the evolution of the autonomous regime of the studied system. It is in this sense that a transformation called homographic [4], or as transformation trapeze [18], this transformation allows to combine the rigorous way to a continuous system a discrete system. According Hung and Chou [7], one may use the trapezoidal rule for integrating the state equation over the interval kT<t< (k+ 1) T to obtain:

$\dot{x}(t) = \dfrac{x(k+1) - x(k)}{T}$: Discrete differentiation.

$x(t) = \dfrac{x(k+1) + x(k)}{2}$: Average value.

$$x(t-h) = \frac{x((k+1)T - qT) + x(kT - qT)}{2} = \frac{x(k+1-q) + x(k-q)}{2}$$: Average value.

Replace $\dot{x}(t)$, $x(t)$ and $x(t-h)$ by their new expressions in the system of equation defined in (1) we have obtained:

$$\frac{x(k+1)-x(k)}{T} = A_0 \frac{x(k+1)+x(k)}{2} + A_1 \frac{x(k+1-q)+x(k-q)}{2} + Bu(k) \tag{10}$$

Solving equation number (10) for $x\,(k+1)$ we obtain:

$$x(k+1) = (I - \frac{T}{2}A_0)^{-1}(I + \frac{T}{2}A_0)\,x(k) + (I - \frac{T}{2}A_0)^{-1}\frac{T}{2}A_1\,[x(k+1-q) + x(k-q)] + (I - \frac{T}{2}A_0)^{-1}TBu(k) \tag{11}$$

Then

$$x(k+1) = \varphi_1\,x(k) + \varphi_2\,[x(k+1-q) + x(k-q)] + \varphi_3\,u(k) \tag{12}$$

Where:

$$\varphi_1 = (I - \frac{T}{2}A_0)^{-1}(I + \frac{T}{2}A_0) \tag{13}$$

$$\varphi_2 = (I - \frac{T}{2}A_0)^{-1}\frac{T}{2}A_1 \tag{14}$$

$$\varphi_3 = (I - \frac{T}{2}A_0)^{-1}TB \tag{15}$$

III. Discretization of Systems with Internal and External Point Delays

Our problem may be stated as the determination of a discrete time approximation corresponding to the following state-space equations:

$$\begin{cases} \dot{x}(t) = A_0 x(t) + A_1 x(t-h_0) + B_0 u(t) + B_1 u(t-h_1) \\ y(t) = cx(t) \end{cases} \tag{16}$$

Where:

$A_0 x(t)$: Original Term state.

$A_1 x(t-h_0)$: Delayed Term state.

$B_0 u(t)$: Original Term control.

$B_1 u(t-h_1)$: Delayed Term control.

With $h_0 = q_0 T$: a multiple delay of the sampling period and q_0 is an integer.

$h_1 = q_1 T$: a multiple delay of the sampling period and q_1 is an integer.

Note: q_0 and q_1 are not necessarily equal.

T: The sampling period is chosen suitably.

x, u *and* y Respectively are the state vector, the vector and the control vector output.

A_0, A_1, B_0, B_1 and C are matrices of suitable dimensions.

A. State-transition method

The concept of a state-transition has been important in many theories. The state-transition matrix can be used to obtain the general solution of linear dynamical systems. It is also known as the matrix exponential. In the time-variant case, there are many different functions that may satisfy these requirements, and the solution is dependent on the structure of the system. The state-transition matrix must be determined before analysis on the time-varying solution can continue. Assuming that, the input is allowed to vary only at the sampling instants and held constant during each sampling interval, we obtain the following equation:

$$x(k+1) = \theta_1\,x(k) + \theta_2\,x(k-q_0) + \theta_3\,u(\mathrm{k}) + \theta_4\,u(\mathrm{k}-\mathrm{q}_1) \tag{17}$$

$$\forall t \in [kT, (k+1)\mathrm{T}], u(t) = u(k) \;\; and \;\; u(t-h_1) = u(k-q_1) \tag{18}$$

Where:

$$\theta_1 = e^{A_0 T} \tag{19}$$

$$\theta_2 = e^{A_1 T} \tag{20}$$

$$\theta_3 = \int_0^T e^{A_0 t}\;B_0 dt + \int_0^T e^{A_1 t}\;B_0 dt \tag{21}$$

$$\theta_4 = \int_0^T e^{A_0 t}\;B_1 dt + \int_0^T e^{A_1 t}\;B_1 dt \tag{22}$$

The matrices θ_1, θ_2, θ_3 and θ_4 can be conveniently calculated on a digital computer using the following series:

$$\theta_1 = I + A_0 T + \frac{A_0^2 T^2}{2!} + \frac{A_0^3 T^3}{3!} + \ldots\ldots \tag{23}$$

$$\theta_2 = I + A_1 T + \frac{A_1^2 T^2}{2!} + \frac{A_1^3 T^3}{3!} + \ldots\ldots \tag{24}$$

$$\theta_3 = \left(2I + \frac{A_0 T}{2!} + \frac{A_1 T}{2!} + \frac{A_0^2 T^2}{3!} + \frac{A_1^2 T^2}{3!} + \ldots \right) TB_0 \tag{25}$$

$$\theta_4 = \left(2I + \frac{A_0 T}{2!} + \frac{A_1 T}{2!} + \frac{A_0^2 T^2}{3!} + \frac{A_1^2 T^2}{3!} + \ldots \right) TB_1 \tag{26}$$

B. *Method based on trapezoidal rule of integration*

This transformation allows to associate rigorous manner to any continuous system, a discrete system. It is possible to deduce a recurrent solution describing state space of the evolution of a vector x (k) obtained by discretization x (t) at sampling instants. Using the approximations adopted in the homographic transformation matrix discrete model that describes the state space equation we obtain:

$\dot{x}(t) = \dfrac{x(k+1) - x(k)}{T}$: Discrete differentiation.

$x(t) = \dfrac{x(k+1) + x(k)}{2}$:Average value.

$$x(t-h_0) = \frac{x((k+1)T - q_0 T) + x(kT - q_0 T)}{2} = \frac{x(k+1-q_0) + x(k-q_0)}{2}$$: Average value.

$$x(t-h_1) = \frac{x((k+1)T - q_1 T) + x(kT - q_1 T)}{2} = \frac{x(k+1-q_1) + x(k-q_1)}{2}$$: Average value.

Replace $\dot{x}(t)$, $x(t)$, $x(t-h_0)$ and $x(t-h_1)$ by their new expressions in the system of equation defined in (16) we have obtained:

$$\begin{aligned} x(k+1) &= (I - \tfrac{T}{2}A_0)^{-1}(I + \tfrac{T}{2}A_0)\, x(k) + \\ &+ (I - \tfrac{T}{2}A_0)^{-1}\tfrac{T}{2}A_1 \left[x(k+1-q_0) + x(k-q_0) \right] + \\ &+ (I - \tfrac{T}{2}A_0)^{-1} TB_0 u(k) + (I - \tfrac{T}{2}A_0)^{-1} TB_1 u(k-q_1) \end{aligned} \tag{27}$$

Then

$$\begin{aligned} x(k+1) &= \sigma_1 x(k) + \sigma_2 \left[x(k+1-q_0) + x(k-q_0) \right] + \\ &+ \sigma_3 u(k) + \sigma_4 u(k-q_1) \end{aligned} \tag{28}$$

Where:

$$\sigma_1 = (I - \frac{T}{2}A_0)^{-1}(I + \frac{T}{2}A_0) \tag{29}$$

$$\sigma_2 = (I - \frac{T}{2}A_0)^{-1}\frac{T}{2}A_1 \tag{30}$$

$$\sigma_3 = (I - \frac{T}{2}A_0)^{-1}TB_0 \tag{31}$$

$$\sigma_4 = (I - \frac{T}{2}A_0)^{-1}TB_1 \tag{32}$$

IV. NUMERICAL EXAMPLES

In this section two different examples of application of the proposed approach are presented. Those examples show the validity of the methodology. As a first example, the next system is considered by the state-space equation:

$$\begin{cases} \dot{x}(t) = \begin{pmatrix} 0 & 1 \\ -2 & -3 \end{pmatrix} x(t) + \begin{pmatrix} 0 & 0 \\ -1 & 2 \end{pmatrix} x(t-0.2) + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u(t) \\ y(t) = (1 \quad 1)\, x(t) \end{cases} \tag{33}$$

In this case we are treating an example of a continuous linear system. A delay is only in the state which was chosen a sampling period T = 0.2s and applying the state-transition method, the following discrete time-delay system is obtained:

$$x(k+1) = \begin{pmatrix} 0.9680 & 0.1493 \\ -0.2987 & 0.5200 \end{pmatrix} x(k) + \begin{pmatrix} 1 & 0 \\ -0.2453 & 1.4907 \end{pmatrix} x(k-q) + \begin{pmatrix} 0.0160 \\ 0.3947 \end{pmatrix} u(\mathrm{k}) \tag{34}$$

The simulation result in the step responses of the continuous system and approximate discrete system is depicted in figure1.

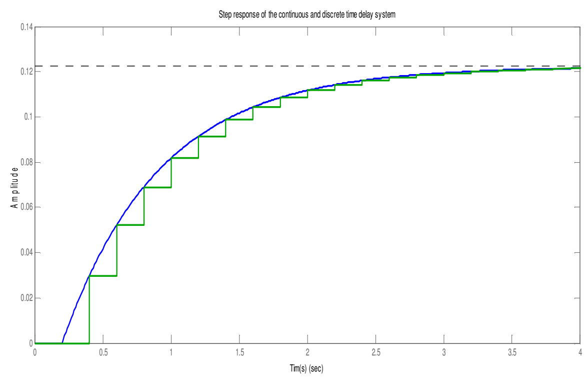

Fig. 1. Step response of the continuous and discrete time delay system.

We note that the system response does not swing; that allows us to conclude that the system is highly damped. Characteristic desired system performance is specified by a delay time, a rise time and a settling time. Thus, very often the performance characteristics of the controlled system are specified in terms of transient response to a step input, easy to generate. The delay in digital control systems is always a challenge to take into consideration. It introduces a phase shift of the signal as it can induce instability. The evolution of the response the field below presents a delay that does not degrade the performance of the system.

Applying the method based on trapezoidal rule of integration to the system considered in the state-space equations (33), and keeping the same sampling period T=0.2s. The following discrete time-delay system is obtained:

$$
\begin{aligned}
x(\mathrm{k}+1) &= \begin{pmatrix} 0.9697 & 0.1515 \\ -0.3030 & 0.5152 \end{pmatrix} x(k) + \\
&+ \begin{pmatrix} -0.0758 & 0.1515 \\ -0.7576 & 1.5152 \end{pmatrix} [x(k+1-q) + x(k-q)] + \\
&+ \begin{pmatrix} 0.0152 \\ 0.1515 \end{pmatrix} u(k)
\end{aligned}
\tag{35}
$$

The simulation result in the step responses of the continuous system and approximate discrete system is depicted in figure2.

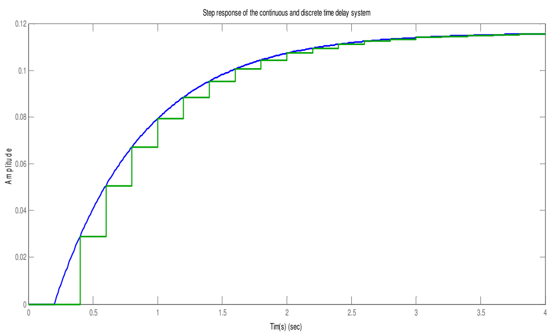

Fig. 2. Step response of the continuous and discrete time delay system.

We note that, the representation of discrete time system, in figure 1 and figure 2; follows the continuous time evolution of the system start after a while which is the value of the existing delay in the state equation that defines our system, keeping almost same stability range in both cases.

In this case, the model is insufficient for an accurate representation of the closed-loop response and a more complex approximate discrete system should be considered in order to improve the result. Precision requirements will determine the number of terms to be considered in the exponential series, depending on the application. Anyway, simple discrete systems can be a useful tool, combined with robust control techniques.

In the next example, one external and one internal delay are considered in the state-space equation:

$$
\begin{cases}
\dot{x}(t) = \begin{pmatrix} -1 & -1.5 \\ -1 & -2.5 \end{pmatrix} x(t) + \begin{pmatrix} 0.25 & -0.5 \\ 0.5 & 0.5 \end{pmatrix} x(t-0.3) + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u(t) + \begin{pmatrix} 0.5 \\ 0.5 \end{pmatrix} u(t-0.5) \\
y(t) = (1 \quad 1) x(t)
\end{cases}
\tag{36}
$$

In this case, the system has a different internal and external delay and, then, combinations of those delays appear in the input of the discrete approximation. Considering T =0.1s and applying the state-transition method. The following discrete time-delay system is obtained:

$$
\begin{aligned}
x(k+1) &= \begin{pmatrix} 0.9112 & -0.1266 \\ -0.0844 & 0.7846 \end{pmatrix} x(k) + \begin{pmatrix} 1.0240 & -0.0519 \\ 0.0519 & -1.0500 \end{pmatrix} x(k-q_0) + \\
&+ \begin{pmatrix} -0.0092 \\ 0.1913 \end{pmatrix} u(\mathrm{k}) + \begin{pmatrix} 0.0937 \\ 0.0947 \end{pmatrix} u(\mathrm{k}-\mathrm{q}_1)
\end{aligned}
\tag{37}
$$

The simulation result in the step responses of the continuous system and approximate discrete system is depicted in figure3.

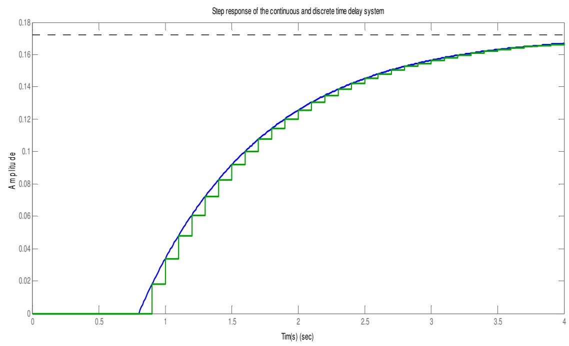

Fig. 3. Step response of the continuous and discrete time delay system.

In practice, the sampling period T_e depends on the type of method (chemical, thermal, mechanical, etc.) and should be chosen small relative to the time of the closed loop system response. Sampling period is too low (sampling) results

• Closer to the continuous time, but the calculation is much more demanding.

• Difficulties inherent in the methods of calculation order, which may lose strength. In the present case the delay is a multiple of the sampling period or of its value is imposed by the delay. We can see very clearly the importance of the sampling period: a lower sampling period provides monitoring of the response of faithful continuous system with a period greater. The consequence of this is that the system with the lowest sampling period is the one who achieve stability quickly. So to have the most accurate possible relative to the continuous time result, it will be advantageous not to take a sampling period too large.

Applying the method based on trapezoidal rule of integration to the system considered in the state-space equations (36), and keeping the same sampling period T=0.1s.The following discrete time-delay system is obtained:

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$$
\begin{aligned}
x(\mathrm{k}+1) &= \begin{pmatrix} 0.9108 & -0.1274 \\ -0.0849 & 0.7834 \end{pmatrix} x(k) + \\
&\begin{pmatrix} 0.2070 & -0.5096 \\ 0.4352 & -0.4671 \end{pmatrix} [x(k+1-q_0) + x(k-q_0)] + \\
&+ \begin{pmatrix} -0.0064 \\ 0.0892 \end{pmatrix} u(k) + \begin{pmatrix} 0.0446 \\ 0.0425 \end{pmatrix} u(k-q_1)
\end{aligned}
\tag{38}
$$

The simulation result in the step responses of the continuous system and approximate discrete system is depicted in figure4.

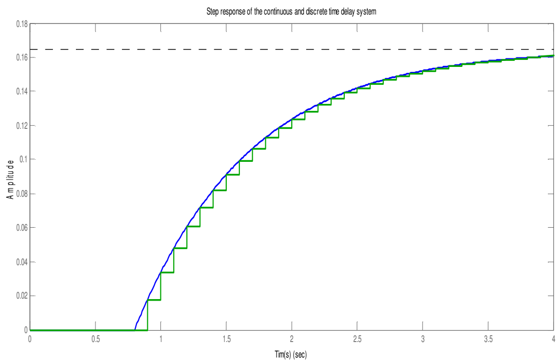

Fig. 4. Step response of the continuous and discrete time delay system.

It can be seen in figure 3 and figure 4, that the representation of discrete time system follows the continuous time evolution of the system start after a certain time which corresponds to the value that is the result of the existing delays in both the state and the control in the equation defined in our system keeping almost the same area of stability, in both cases. But it could not present the same response time and the appearance of a second term; describing the past state of the system during a second sampling instant by the discretization homographic transformation method that requires computation of time additive. The effect of delay on the dynamics of a system depends not only on its value but also the characteristics of the system. Indeed, the presence of delay can affect the stability of the system, since its presence can cause complex behaviors namely oscillations, instability and degradation in performance. But sometimes the effect of the delay in some cases can be stabilizing an unstable system initially. In this case, we say that the delay has a stabilizing effect.

V. Comparison between two methods

We have discussed two different methods for obtaining discrete time approximations for continuous-time delay systems. The state-transition method has much more complications for calculating than the homographic transformation method. Suggesting that the sampling period should be chosen over the dynamic system, depending on the delay term. Generally we try to choose the sampling period as an integer multiple of the delay in addition to the forced choice for dynamic system without delay. It appears that the model using the homographic transformation is best suited for numerical simulation in the case of the continuous system which is defined by a state model and that must be selected in the case of a suitable choice of the sampling period.

It is clear that the value of the delay (0.2s) is shown by the simulation by both discretization methods knowing that it did not affect the evolution of the system. Similarly we see that in the second example there or delays are present in the state (0.3s) and in the control (0.5s) simulation shows that the amount of delay in the system is the result of two delays (0.3s +0.5 s) with a normal evolution of the dynamic system in time without losing performance.

We note that depending on the value of the delay h, the election of this value as sampling period can be inadequate in the sense of the Shannon (Discretization) Theorem. Then, the sampling period can be chosen $T = h/j$ for some integer j, where T is a valid value. In general, considering systems given by equations (33) or (36) and $T = h/j$, the approximate discrete system is given by the two methods.

VI. Conclusion

In this paper we have proposed two methods for the discrete time approximation of multi variable continuous-time delay systems. If the model is given in the form of state equations, the trapezoidal rule requires only one matrix inversion, and is perhaps the most convenient for digital simulation. It is of interest to compare them for accuracy as well as convenience in computation. It is evident that the accuracy of the approximation will depend, to a large extent, on the choice of the nature of the input applied to the delay continuous linear system. For example, if the input is a constant or a piece-wise-constant function of time, the state-transition method will give an accurate discrete time model. So that the discretization of the continuous delay either by the method of transition state or by the method of homographic transformation matrix linear system show that the two examples follow the curve form almost continuously, except that the first method has many more complications than the second calculation, suggesting that the sample period should be chosen in addition to the dynamics of the system, depending on the delay term. Usually we try to choose the sampling period an integer sub multiple of delay in addition to its forced choice for dynamic systems without delay. We note that despite the variation of the period allowed in the simulation curves obtained by the two methods are similar domain discretization and the method using homographic transformation is easier to program. Finally it appears that the model using the homographic transformation is best suited for numerical simulation in the case of the continuous system is defined by a state model and in the case of a suitable choice of the sampling period.

REFERENCES

[1] B. LEE AND J. G. LEE.: Delay-dependent stability criteria for discrete-time delay systems. Proc. Of American Control Conference, pp. 319-320,(1999).

[2] EL MOUDNI A.,BEN SASSI B.: On a new order reduction method for discrete systems.12 th world congress on scientific computation IMACS,pp 95-99,(1988).

[3] G.F. FRANKLIN, J.D. POWELL, M.L. WORKMAN.: Digital Control of Dynamic Systems, Addison-Wesley. New York , (1998).

[4] HAYKIN, S.S.: A unified treatment of recursive digital filtering. IEEE Trans.AC-17, pp. 113-116,(1972).

[5] HUNG, J.C., LIU, C.C., and CHOU, P.Y.: An algebraic method for system parameter identification. Proc. 14th Asilomar Conference on Circuits, Systems and Computers,pp. 277-279,(1980).

[6] HARMUTH, H.F.: Applications of Walsh functions in communications. IEEE Spectrum, 6, pp. 82-91, (1969).

[7] J. JUGO.: Discretization of continuous time delay stems. 15th Triennial Word Congress, D. Electricidad y electronica, F. de Ciencias, UPV/EHU , Apdo. 644, Bilbao, Barcelona, Spain IFAC, (2002).

[8] KAILATH,T. : Linear systems. Prentice Hall, In Proc. IEEE Conf. on Dec. & Contr. pp. 960--961, Albuquerque, New Mexico, (1980).

[9] MARTENS, H.R.: A comparative study of digital integration methods. ibid.12, pp. 87-94, (1969).

[10] NORMEY-RICO AND DANIEL LIMON,E.:Control of dead-time processes. Springer Verlag 50th IEEE Conference on Decision and Control and European Control Conference (CDC-ECC)Orlando, FL, USA, pp 12-15,(2011).

[11] N. K. SINHA AND ZHOU QI-JIE.: Discrete time approximation of multivariable continuous time systems. IEE PROC. Vol. 130, Pt. D, N° 3,(1983).

[12] PALANISAMY, K.R., and BHATTACHARYA, D.K.: System identification via block-pulse functions. Int. J. Syst. Sci,12, pp. 643-647, (1981).

[13] RAO, G.P., and SRINIVASAN, T.: Remarks on author's reply to comments on Design of piecewise constant gains. IEEE Trans, AC-23, pp. 762-763,(1978).

[14] SANNUTI, P.: Analysis and synthesis of dynamic systems via block-pulse functions. Proc. TEE, 124, pp. 569-571 , (1977).

[15] S.MAGDI MAHMOUD.:Robust Filtering for Time-Delay Systems. Marcel Dekker,Elsevier Science Inc. New York, NY, USA.An international journal,Vol 176,Isuue 2,pp.186-200,(2006).

[16] SINHA N.K AND ZOU Q. :Discrete time approximation of multivariable continuous time systems. IEE Proceedings, Vol 130,Pt.D,N°3 ,pp 103-110,(1983).

[17] SINHA, N.K., and LASTMAN, G.J.: Transformation algorithm for identification of continuous-time multivariable systems from discrete data. Electron. Lett, 17, pp. 779-780,(1981).

[18] SHIEH, L.S., YEUNG, C.K., AND MCLNNIS, B.C.: Solutions of state-space equations via block-pulse functions. Int. J. Control, 28, pp. 383-392,(1978).

[19] SINHA, N.K.: Estimation of transfer functions of continuous-time system from sampled data. Proc. IEE, 119, pp. 612-614, (1972).

[20] SINHA, N.K., and ROZSA, P.: Some canonical forms for linear multivariable systems. Int. J. Control, 23, pp. 865-883,(1976).

[21] W. MICHIELS , V. VAN ASSCHE, AND S.-I. NICULESCU .: Stabilization of time-delay

[22] systems with a controlled time-varying delay and applications .IEEE Transactions on Automatic Control, vol. 50, no. 4, pp. 493–504,(2005).

[23] W. MICHIELS AND S.-I. NICULESCU . : Stability and Stabilization of Time-Delay Systems. An Eigenvalue-Based Approach, ser. Advances in Design and Control. vol. 12., SIAM, (2007).

[24] QING-CHANG ZHONG.: Robust Control of Time-delay Systems. IEEE Transactions on Automatic Control ,53, pp. 636-637, (2008).